How does the Black-Litterman Model Calculate Return Forecasts?

Several techniques for creating better return forecasts for use with mean-variance optimization have been developed recently. We believe that we have found the best of these solutions and we have incorporated it into AllocationADVISOR. The Black-Litterman forecast model creates return forecasts which are based on sound economic theory and which help harness the power of mean-variance optimization. Using Black-Litterman return forecasts in mean-variance optimization results in intuitive, diversified portfolios which are relevant for practical investing. For more information about how Black-Litterman leads to diversified portfolios, see the article "Black-Litterman: Asset allocations you can actually use!" located on our website at: http://www.styleadvisor.com/resources/newsletters/BLPortfolios.html

How does the Black-Litterman method create return forecasts?

The model uses a technique called reverse optimization to determine the *Implied Returns* of a portfolio based on the available market capitalization of the asset classes being optimized. It also provides a framework to mix investor views with the Implied Returns to form a new combined estimate of returns.

Implied Returns

Black-Litterman return forecasts are based on the Implied Returns. Implied Returns is a concept which is based on market equilibrium. The investor first selects an asset palette, which is the set of assets that will be optimized. This asset palette is assumed to be the market. We further assume that the market is in equilibrium. "Equilibrium" really means that the market price is such that supply equals demand. In this case, we are assuming that the supply of assets is equal to the demand for the assets. When we are talking about assets, the price becomes a return. It is the return that is implied by the market equilibrium that we want to find. This is the Implied Returns.

What is the market equilibrium? If we believe in efficient markets, then the market today is in equilibrium. The equilibrium portfolio, then, is the market portfolio. The market portfolio is the capitalization-weighted portfolio of the assets.

Most of the time investors will use multiple asset classes when creating an asset allocation. In order to demonstrate the mathematics behind the model without resorting to matrix algebra (which is beyond the scope of this article) we will look at a simple three asset example:

Asset	*Market Cap*
US Equity	$ 11,498
US Bonds	$ 8,280
Int'l Equity	$ 10,350

The Implied Returns, as the name suggests, are the returns which are implied by the cap-weighted market portfolio. The Implied Returns are calculated using reverse optimization. This is sometimes referred to as "backing out" the returns. There are three components of the

calculation of Implied Returns, the Risk Aversion Coefficient, the Covariance Matrix and the Market Portfolio Weights.

To start, we will calculate the Implied Excess Returns. At the end we will convert these to Total Returns. In order to keep this article as user-friendly as possible, we will spell out as much as possible, and avoid using symbols.

Implied Excess Returns = Risk Aversion Coefficient * Covariance * Market Capitalization Weights

Let's look at the three components of this calculation.

The Risk Aversion Coefficient (RAC)

The Risk Aversion Coefficient (RAC) is the rate at which more return is required for more risk. It is the palette risk premium divided by the variance of the asset palette. The variance is calculated using the historical returns for the assets. The risk premium is entered by the user.

$$\text{Risk Aversion Coefficient} = \frac{\text{Risk Premium}}{\text{Variance}}$$

The forward looking Risk Premium is one of the most contentious topics in finance. The *equity* Risk Premium is the expected excess return of equity over the Risk-Free Rate. For the calculation of the Implied Returns, the Risk Premium is an estimate of the Asset Palette's excess return over the Risk-Free Rate. The Asset Palette Risk Premium acts as a scaling factor in the reverse optimization process. While we recommend that you select a reasonable number, the actual number (assuming it is positive) will not change the composition of the efficient allocations that form the efficient frontier. What is affected by the Risk Premium is the magnitude of the return forecasts. An unrealistic Risk Premium results in unrealistic forecast returns leading to unrealistic conclusions regarding future wealth.

For our three asset example we will forecast a Risk Premium of 4%. The Variance of the Asset Palette is 1.117%.

$$\text{Risk Aversion Coefficient} = \frac{4.00\%}{1.117\%} = 3.404$$

Covariance (Cov)

The covariance (COV) measures the correlation in the fluctuation of the return series. The most common example of this is the performance of equity and fixed income. It is generally accepted that when equity is performing well, fixed income yields tend to be lower. Conversely, when equity is not performing well, fixed income yields are higher. The covariance captures this relationship between assets.

The covariance of each pair of assets is calculated using historical correlations and standard deviations. So, the covariance of Assets A and B is:

Covariance (A,B) = Correlation (A,B) * Standard Deviation (A) * Standard Deviation (B)

As an example, we will look at the covariances for US Equity:

Cov(*US Equity, US Equity)* = 0.036

Cov(*US Equity, US Bonds)* = 0.002

Cov(*US Equity, Int'l Equity)* = 0.010

The other covariances are calculated, and the following covariance matrix is formed:

Covariance	***US Equity***	***US Bonds***	***Int'l Equity***
US Equity	**0.036**	**0.002**	**0.010**
US Bonds	**0.002**	**0.003**	**0.001**
Int'l Equity	**0.010**	**0.001**	**0.025**

Market Portfolio Weights (MPW)
For the calculation of the Implied Returns we are assuming that the market is in equilibrium. The equilibrium portfolio is the Market Portfolio. The weights of each of the assets in the Market Portfolio are calculated using the market capitalization of each of the assets. The weight given to each asset is proportional to the asset's share of the total market cap of the Market Portfolio. These weights are called the Market Portfolio Weights.

In AllocationADVISOR, users can either select to use assets for which we provide monthly market cap estimates, or create their own data series with a market capitalization value.

For our three assets, the market caps and weights look like this:

	Market Cap	***MPW***
US Equity	$ 11,498	38.2%
US Bonds	$ 8,280	27.5%
Int'l Equity	$ 10,350	34.4%
TOTAL	**$ 28,980**	**100%**

As an example, we can look at the calculation of the weight for US equity:

$$MPW_{USEquity} = \frac{\text{US Equity Market Cap}}{\text{Total Market Cap}} = \frac{11.498}{28.980} = .382$$

Implied Excess Returns (IER)
Now that we have the three necessary pieces, let's put them together and calculate the Implied Returns. Due to the nature of the covariance matrix-the second element in the formula-the calculations are generally made using matrix algebra. It is for this reason that we have chosen

to use a three asset example in this article. We can now break down the formula for Implied Excess Returns without having to resort to matrix algebra:

$$IER_{USEquity} = RAC * [\ Cov(\text{US Equity, US Equity}) * MPW_{US\ Equity} + Cov(\text{US Equity, US Bonds}) * MPW_{US\ Bonds} + Cov(\text{US Equity, Int'l Equity}) * MPW_{Intl\ Equity}\ ]$$

$$= 3.404 * [.036*.382 + .002*.275 + .010*.344] = \mathbf{6.05\%}$$

Implied Returns

The final step is to turn this Implied Excess Return into a Total Return. To do this, the investor must make an estimate of the risk-free rate of return. Here, we will use a risk-free rate of 4.5%.

Implied Return = Risk-Free Rate + Implied Excess Return

Implied Return for US Equity = 4.50 + 6.05 = 10.55%

The Implied Return of the other assets is calculated in the same way.

	Implied Return
US Equity	10.6 %
US Bonds	5.2 %
Int'l Equity	8.9 %

These implied returns can now be used as return forecasts for mean-variance optimization. The maximum Sharpe ratio portfolio on an efficient frontier created using Implied Returns is the market portfolio as you defined it. The fact that the Black-Litterman model recommends holding the market portfolio if you do not have Views is, from a theoretical standpoint, very appealing. Movements away from market capitalization weighted holdings are based on Views.

Views

The Implied Returns are excellent forecasts for use with mean-variance optimization. Investors, however, often have their own opinions about how the market is going to behave in the future. These investors often want to adjust the Implied Returns so that the forecasts better reflect their opinions on future performance.

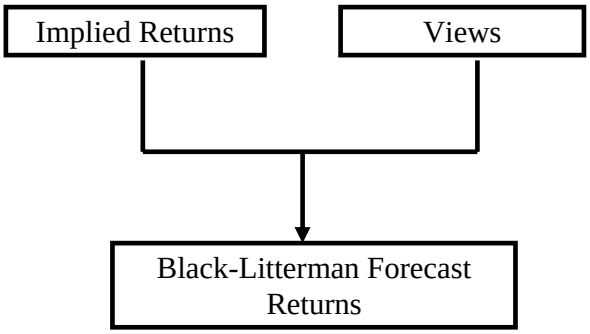
The Black-Litterman method takes an opinion such as "I think that US Equity is going to do well," and quantifies it into something called a View. For this Absolute View, what this really means is that US Equity is going to do better than the 10.6 % forecasted in the Implied Returns. The user must decide how much better US Equity will perform, and assign a level of confidence to the View. The View may then be "I believe with 75% confidence that US Equity's return will be 11.5%."

Users can also create relative Views. The opinion might therefore be "Int'l Equity will outperform US Equity." The Implied Returns forecast the opposite, that US Equity will outperform Int'l Equity. Again, the user must enter the amount of out-performance and a confidence level. An example is "I believe with 85% confidence that US Bonds will outperform US Equity by 1%."

How are these views incorporated into the return forecasts? The Black-Litterman model uses a Bayesian approach to incorporating Views into the forecasts while maintaining the advantage of diversification which comes from using the Implied Returns.

The mathematics of Bayesian probabilities is complicated, but the idea behind it is fairly straightforward. Bayesian probabilities were designed to incorporate subjective beliefs into probability distributions. The user starts with a belief, called the Prior distribution. There is then some event which provides more information, causing the user to wish to modify the distribution. The event is incorporated into the Prior to form the new belief, the Posterior Distribution.

When creating return forecasts for mean-variance optimization, the Prior and Posterior distributions are return distributions. The Prior distribution is the Implied Return distribution. The "events" that are incorporated into the Prior distribution are the investor's Views. The new combined return distribution is the Posterior Distribution. These are the returns that are used as forecasts for mean-variance optimization.



For those of you who would like to know more about the mathematics of the Black-Litterman model and our implementation of it, go to the Black-Litterman Forecast Methodology section in the AllocationADVISOR manual. This section of the manual explains reverse optimization, the process of creating Market Cap Assets, and the Zephyr Asset Palettes. The AllocationADVISOR manual is located in your Style folder as well as the Start Menu StyleADVISOR Program Group.

For the mathematically inclined, a copy of "A Step-By-Step Guide to the Black-Litterman Model: Incorporating User-Specified Confidence Levels" is available upon request (support@styleadvisor.com).